Preventing Unraveling in Social Networks: The Anchored k-Core Problem

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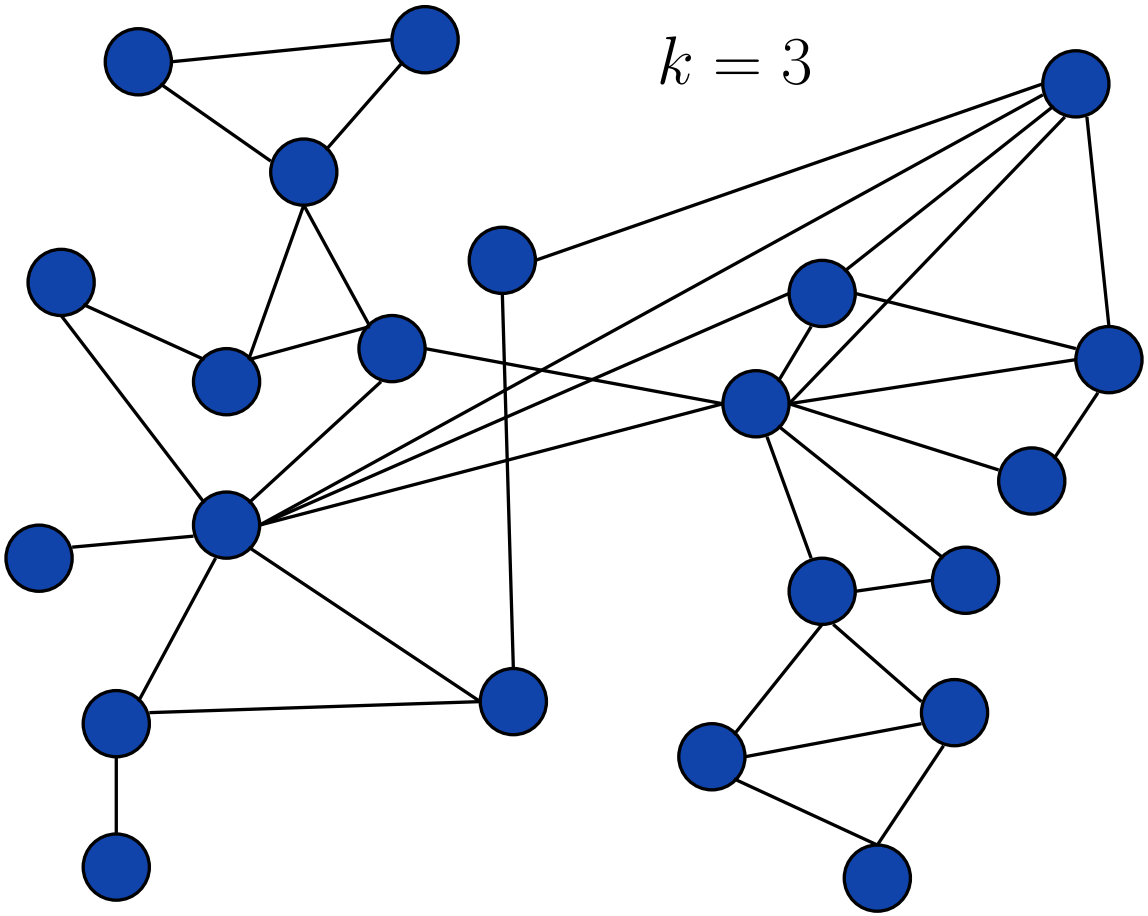
Outline

Anchored k-Core

- The k-core of a graph = the maximal induced subgraph where all vertices have degree at least k.
- Computing the k-core of a graph is easy
- for example, suppose $k = 3$, and G looks like this:

An Example



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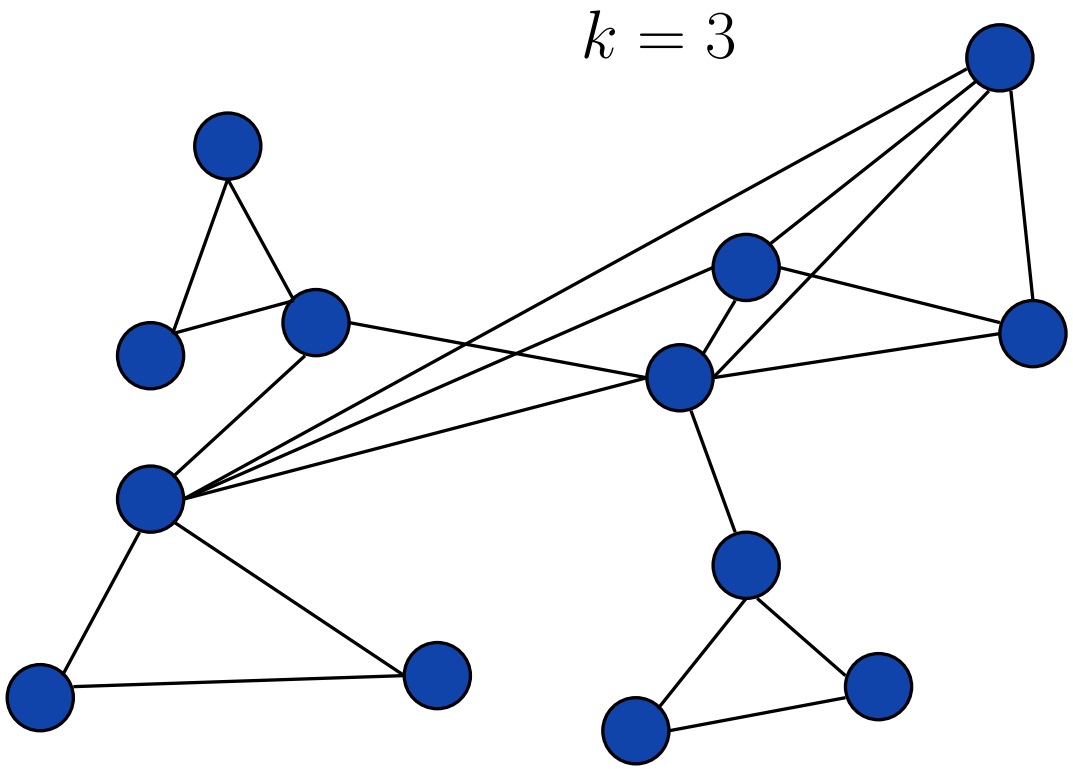

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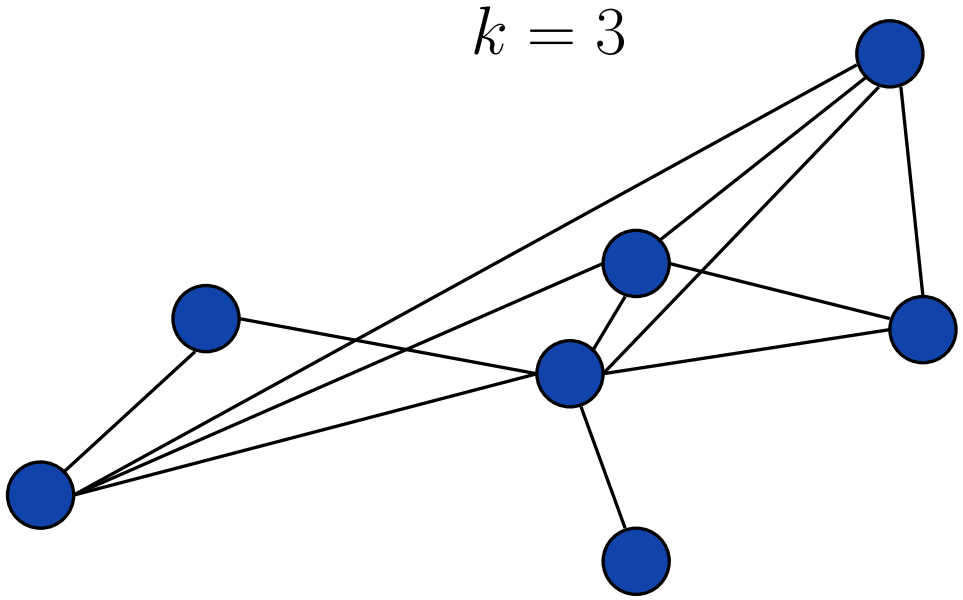

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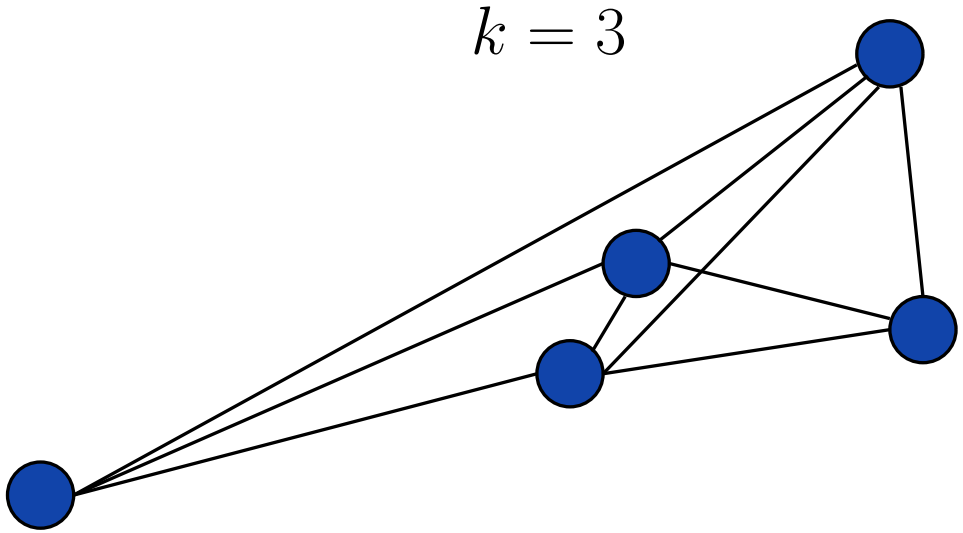

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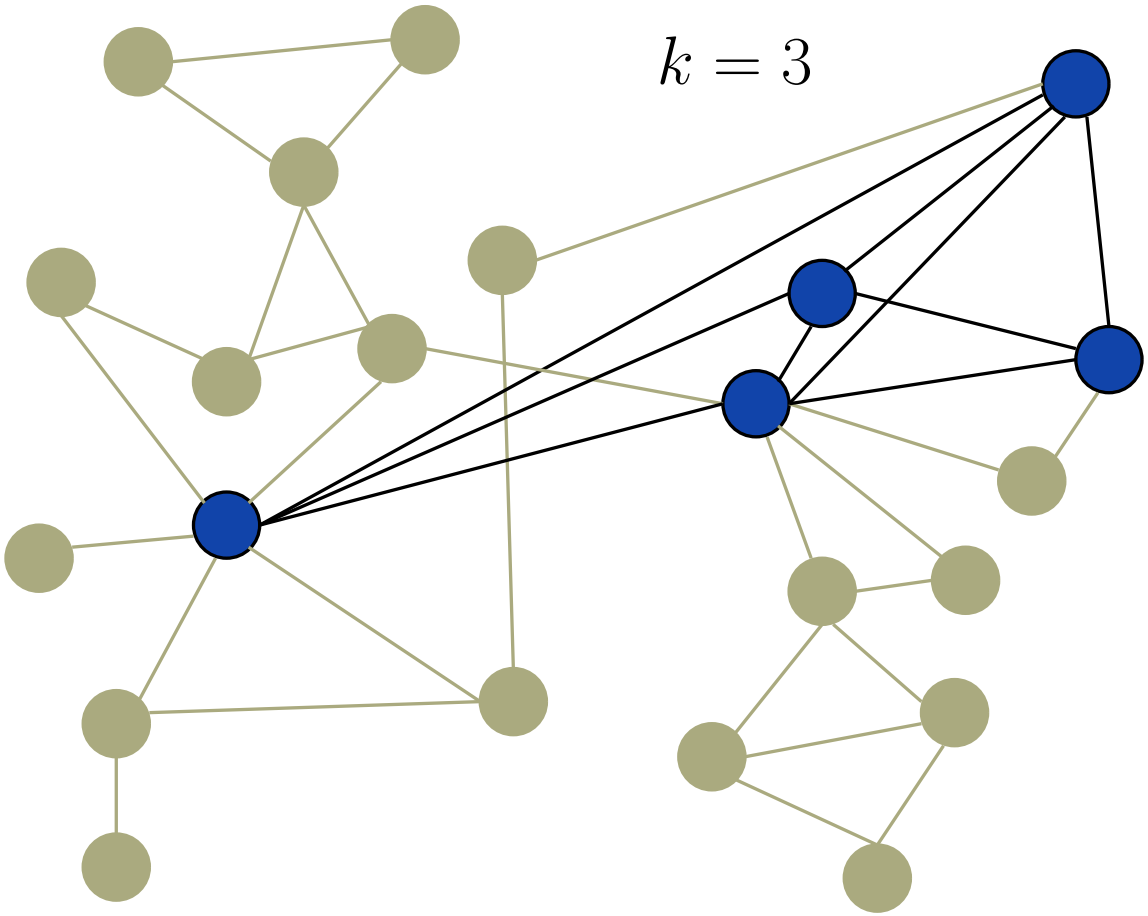

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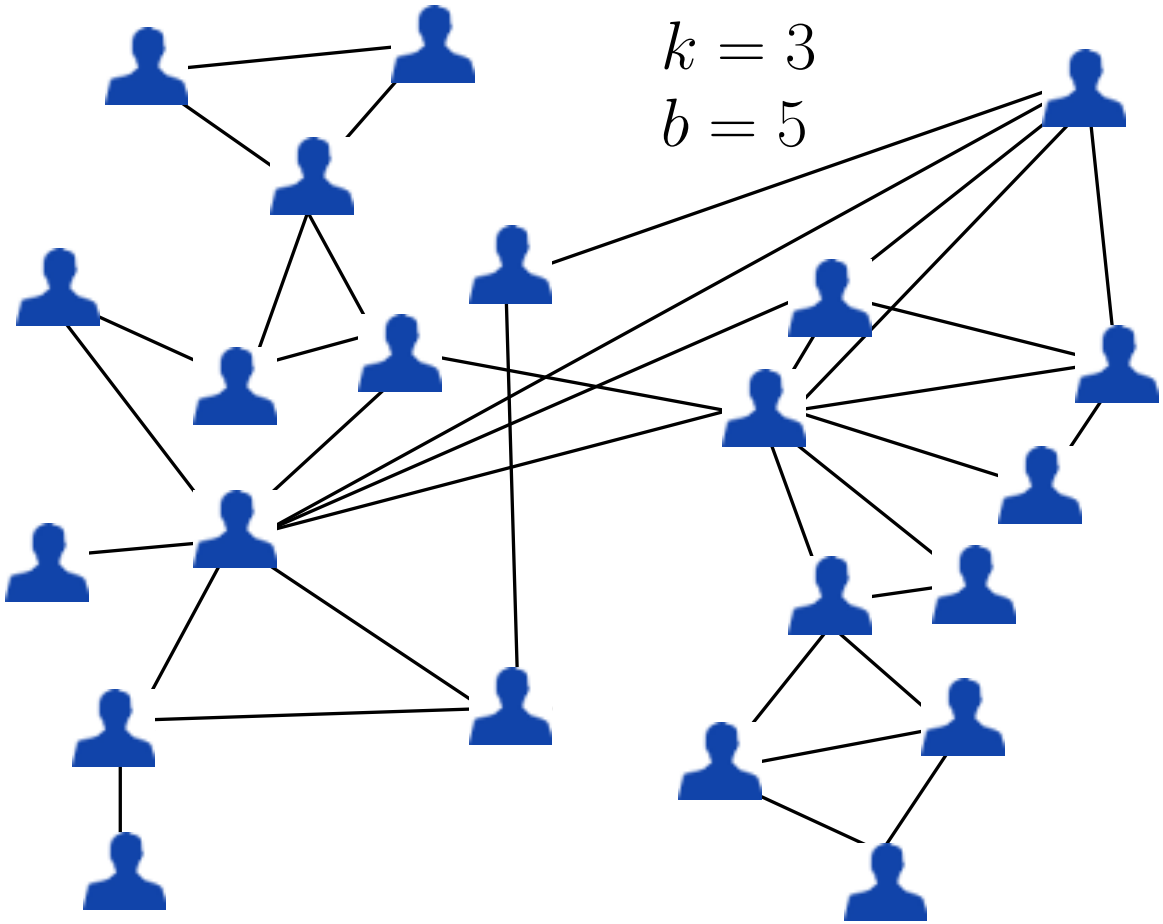
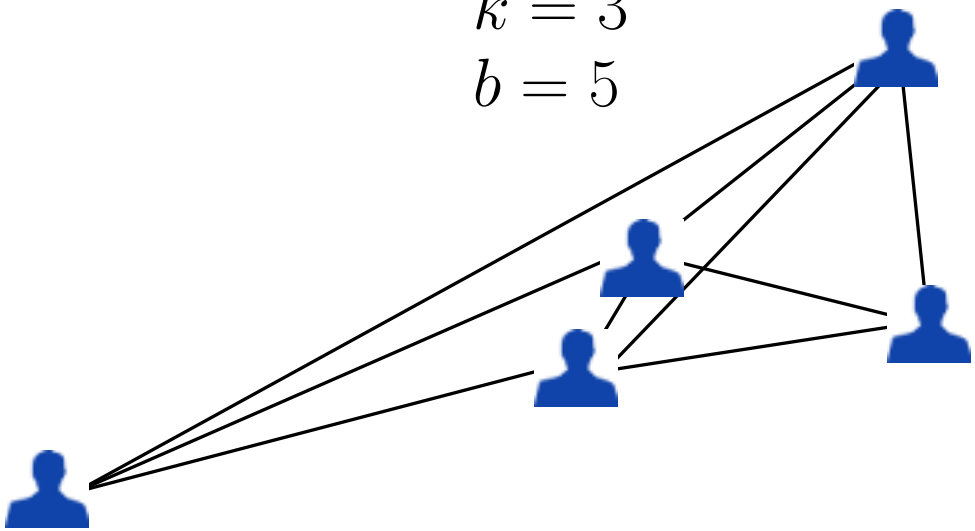
- My goal is to maximize the number of people paying attention
- Also, assume that I have \$50 to spare.

Another Example



Another Example

$k = 3$
$b = 5$



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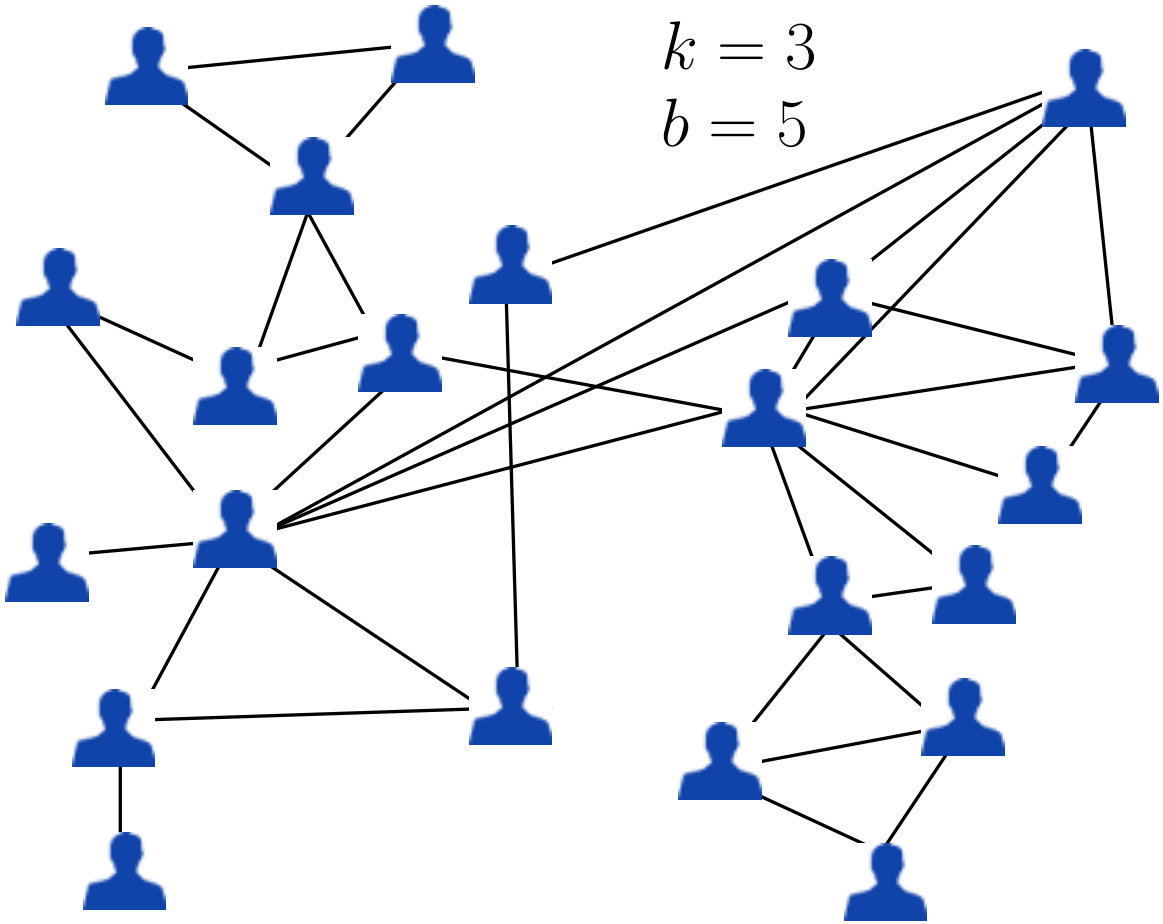

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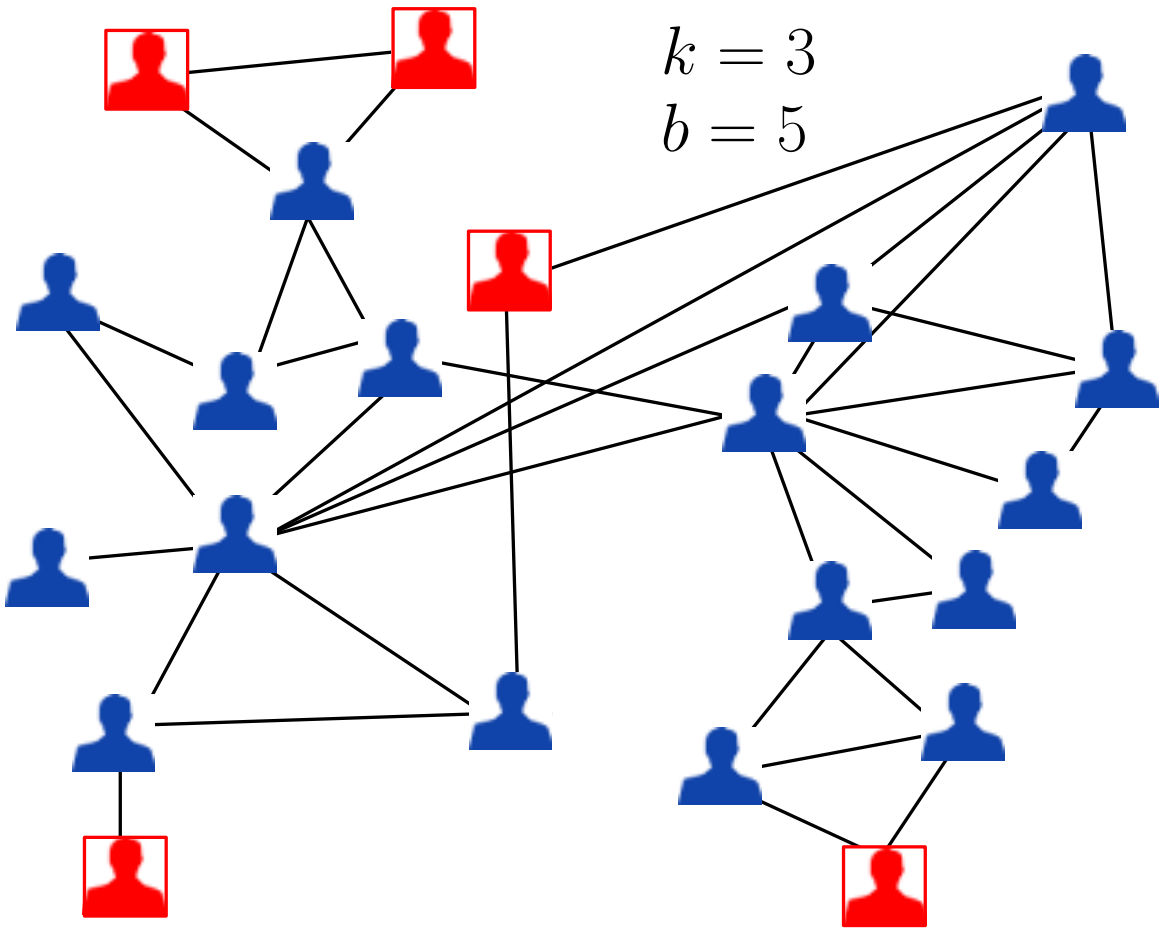

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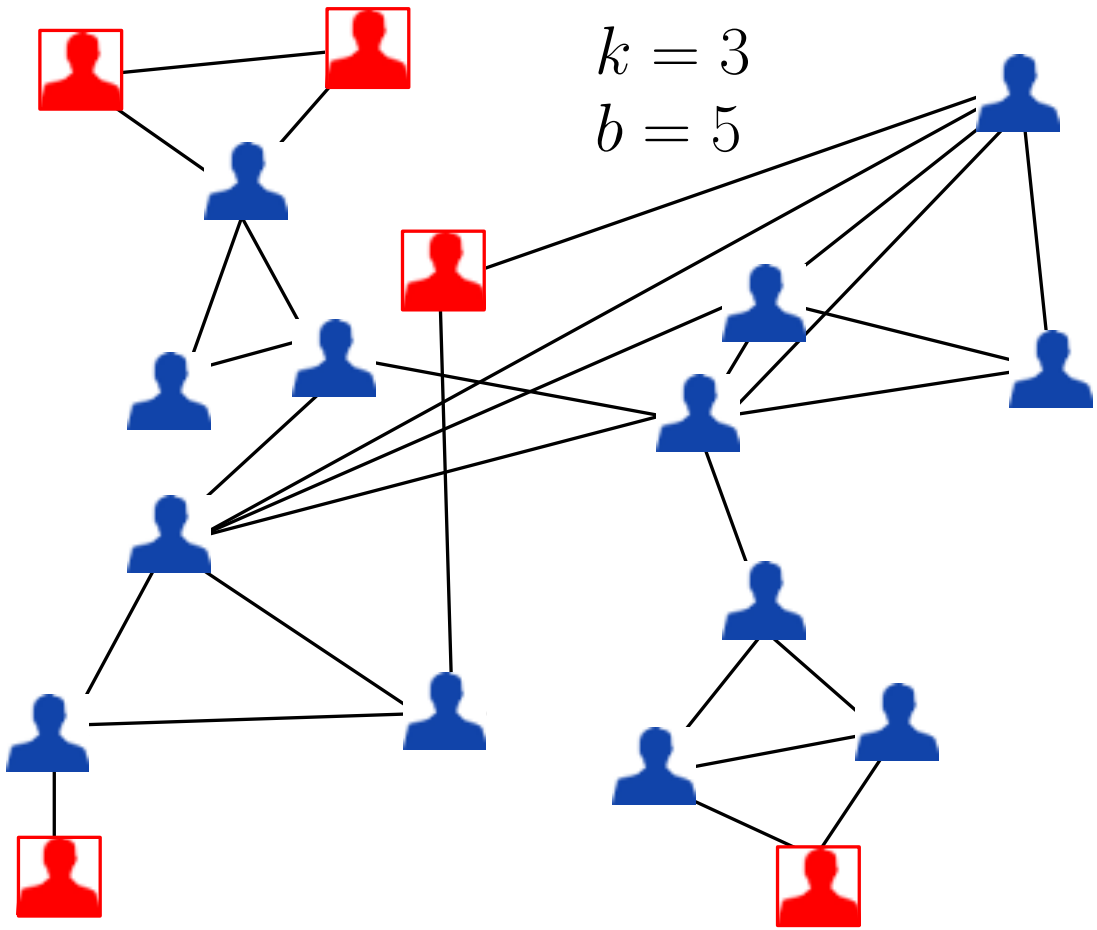

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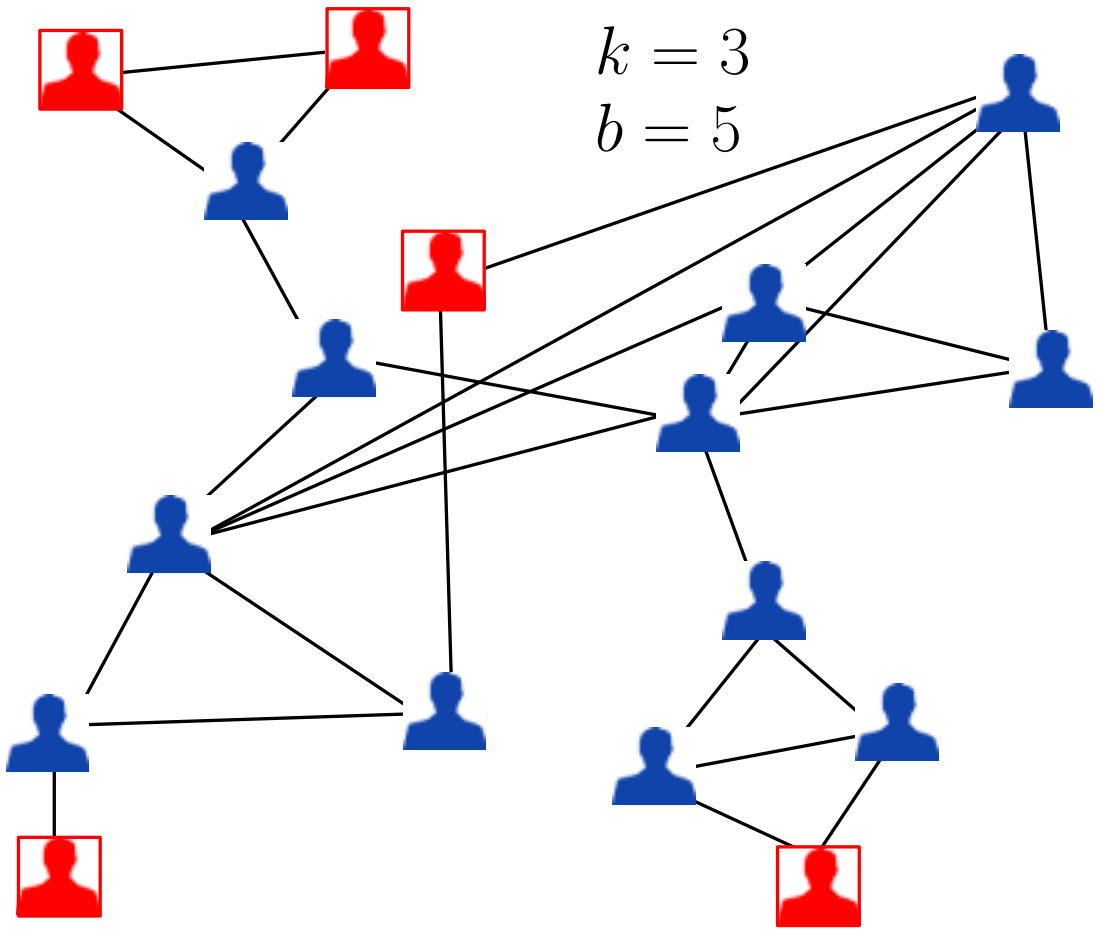

Anchored k-Core with budget b

- the maximum induced subgraph in which every vertex either has degree at least k, or is anchored
- Goal: Find a set S, $|S| = b$, such that placing anchors on the nodes of S maximizes the number of vertices that remain.
- Fact: there is a simple algorithm that runs in time $O(n^b)$
- Fact: $k = 0$ and $k = 1$ are trivial

Anchored 2-Core

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- Observation: Every vertex that lies on a cycle already belongs to the 2-core.
- Algorithm: take longest paths (runs in time $O(n^2)$)
- Can be more clever (runs in time $O(m + n \log n)$)

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Anchored 3-Core

- What happens for $k = 3$?
- Cannot simplify graph into a forest in this case
- In fact, we show that for $k \geq 3$, the problem is difficult.

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- Fact: $b \leq$ solution quality $\leq n$
- So, any algorithm obtains an n/b approximation...

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- Fact: $b \leq$ solution quality $\leq n$
- So, any algorithm obtains an n/b approximation...
- Theorem: It is NP-hard to obtain an $O(n^{1-\epsilon})$ approximation for $k \geq 3$.

Reduction from Set Cover

- Recall Set Cover: n sets, m elements
- covering $< m$ elements is just like covering 0 elements
- this property is ensured by having lots of cycles in the graph
- it's too difficult to figure out how to cover all m elements (because set cover is NP-hard)

Treewidth

- The treewidth of a graph is a measure of how closely a graph resembles a tree
- Trees have treewidth 1, and cliques on n vertices have treewidth $n - 1$

FPT w.r.t. treewidth

- For graphs of treewidth w, we have an $O((3(k+1)^2)^w \cdot b^2 \cdot \text{poly}(n))$-time exact algorithm
- given graph G, decomposes it into a tree
- runs a dynamic programming procedure on the resulting tree

Generalizations

- weighted budget costs
- weighted rewards
- directed edges
- variable thresholds
- variations on the objective function

Summary and Open Problems

- In general, anchored k-core is very difficult, but as soon as there is some structure to the graph, it becomes easier
- PTAS for graphs which exclude fixed minors (planar, etc.)?
- Can it be analyzed for random graphs?

Cheating

Resource Augmentation:

- compare us against the optimal solution using b anchors
- But, allow ourselves to use $2b$ anchors

Cheating

Resource Augmentation:

- compare us against the optimal solution using b anchors
- But, allow ourselves to use $b \cdot \log n$ anchors

Cheating

Resource Augmentation:

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Cheating

Resource Augmentation:

- compare us against the optimal solution using b anchors
- But, allow ourselves to use $b \cdot \text{polylog} n$ anchors
- Still NP-hard to get an $O(n^{1-\epsilon})$ approximation!

Parameterization by the budget

- Is anyone here a fan of FPT?
- Recall that $O(n^b)$ is easy to obtain. Can we get $O(f(b) \cdot \text{poly}(n))$?

Parameterization by the budget

- Is anyone here a fan of FPT?
- Recall that $O(n^b)$ is easy to obtain. Can we get $O(f(b) \cdot \text{poly}(n))$?
- No. The problem is W[2]-hard with respect to b (via Dominating Set).